Lecture 13: Basic Concepts of Wavefront Reconstruction

Astro 289

Claire Max

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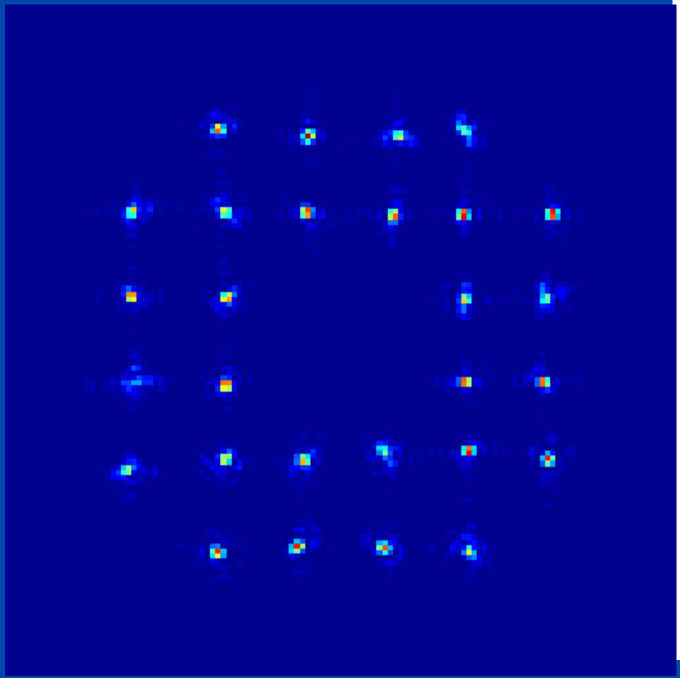
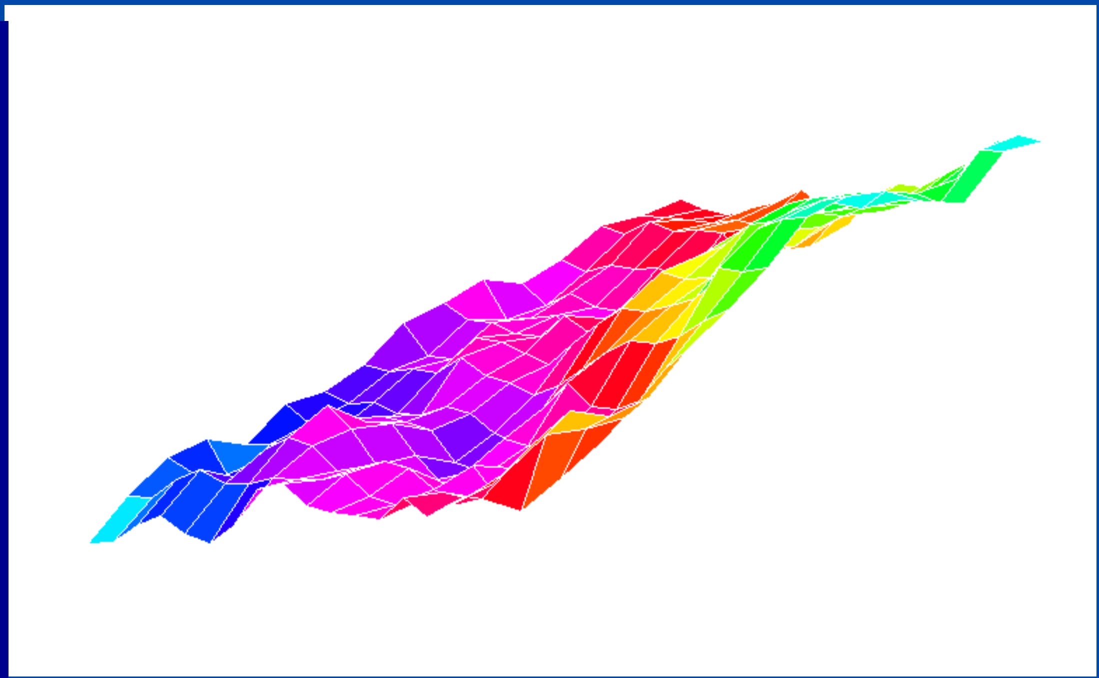
Based on slides by Marcos van Dam and Lisa Poyneer
CfAO Summer School

Outline

- System matrix, H: from actuators to centroids $s = Ha$
- Reconstructor, R: from centroids to actuators $a = Rs$
- Hardware

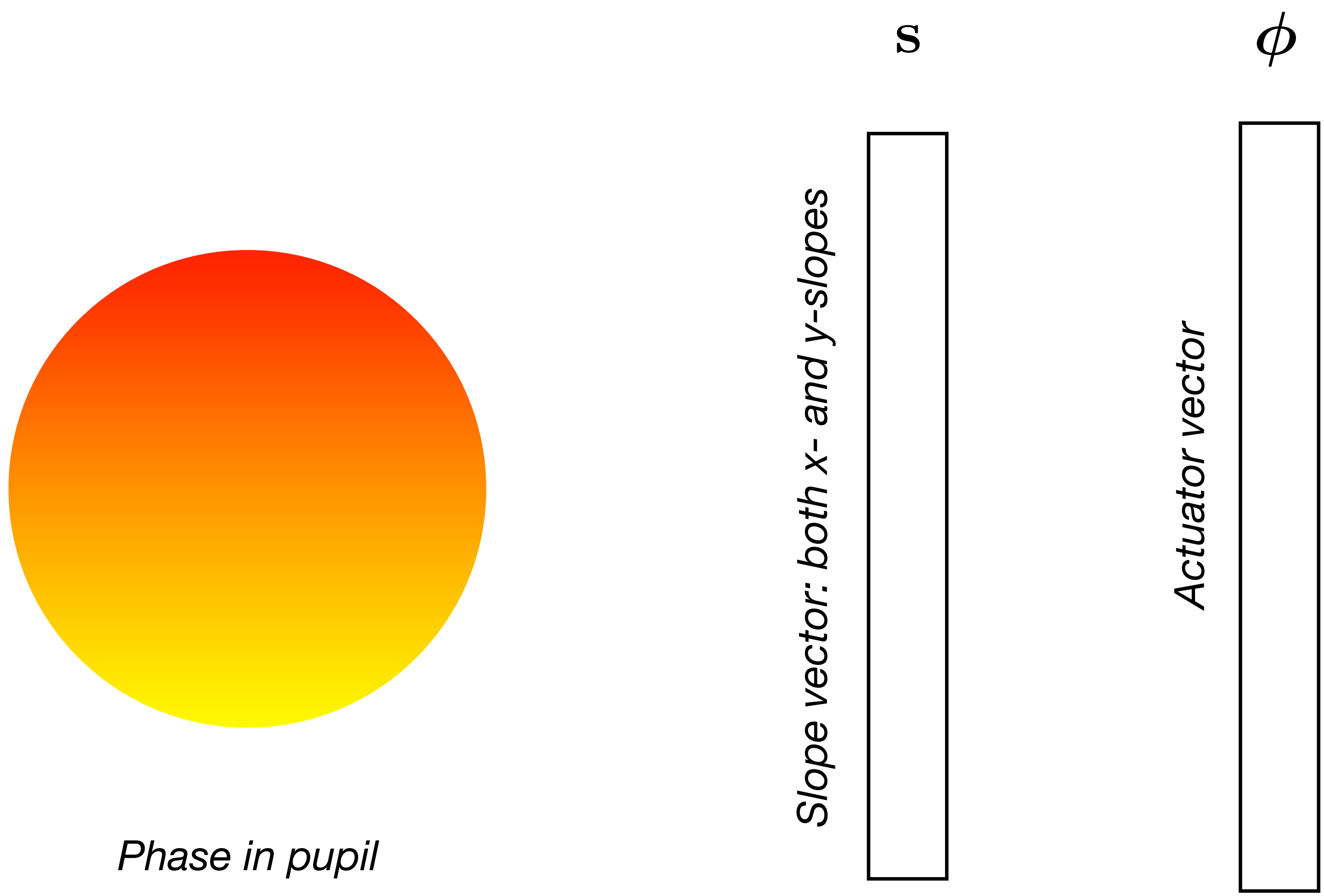


Wavefront sensor



Wavefront (phase)

Mapping subapertures and actuators



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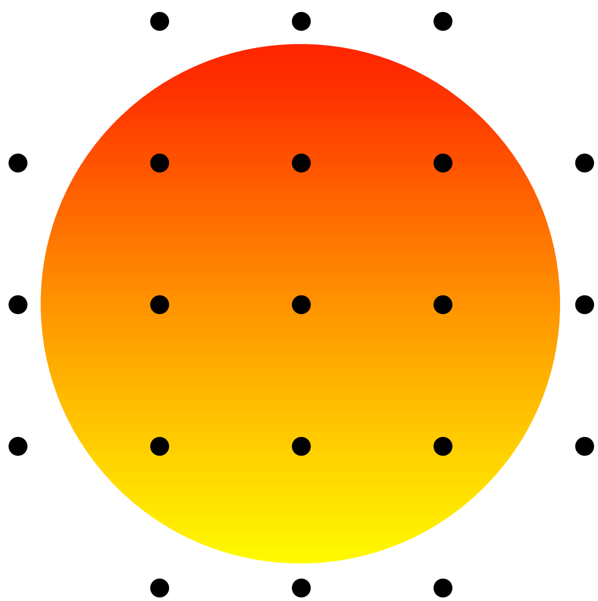
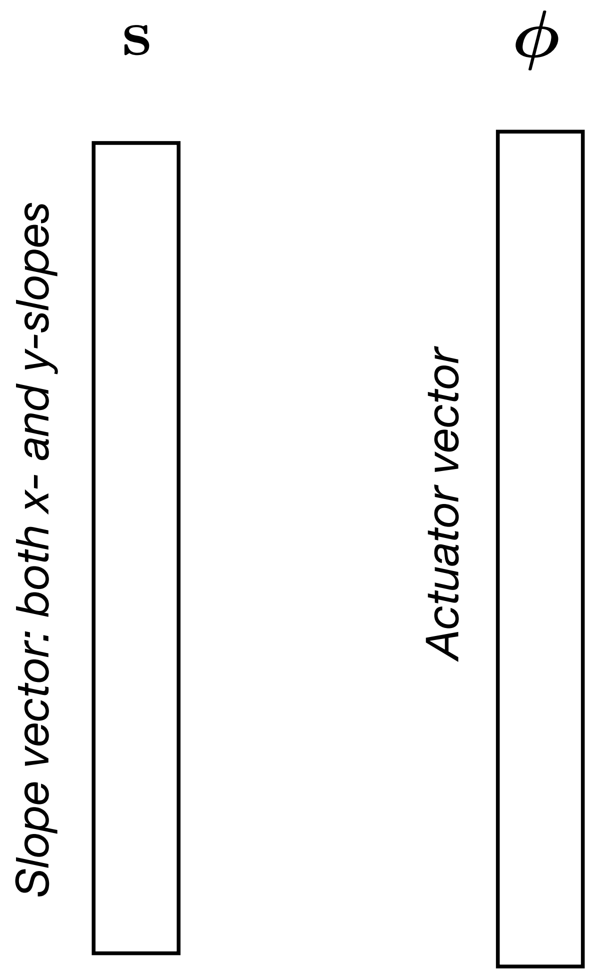


Phase in pupil



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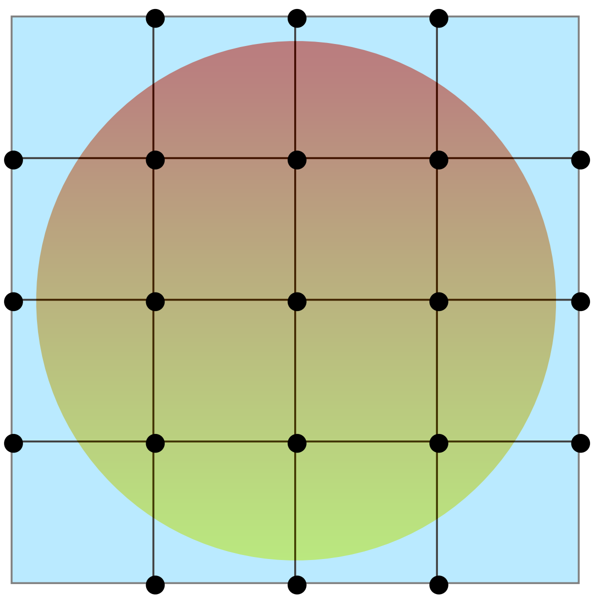
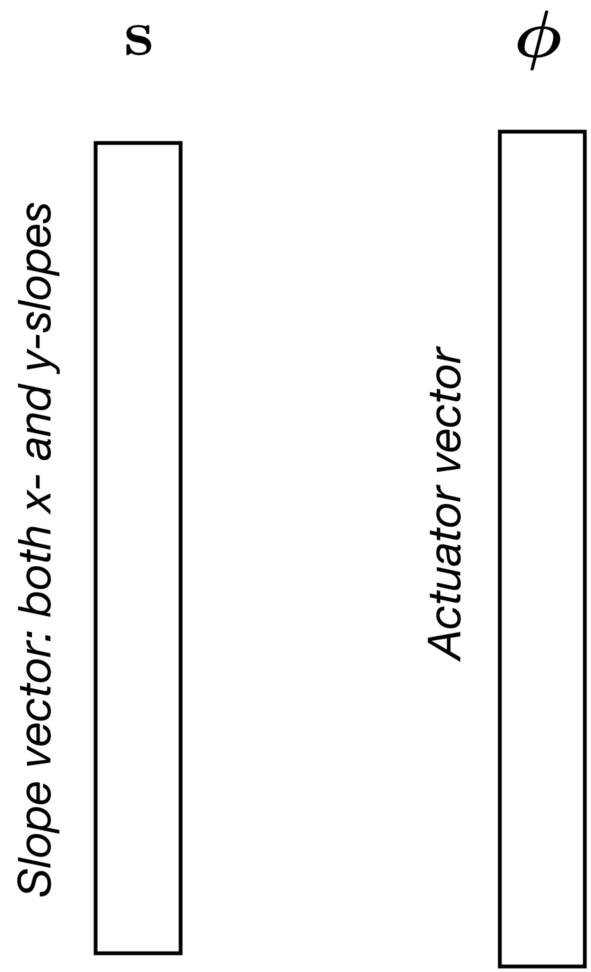
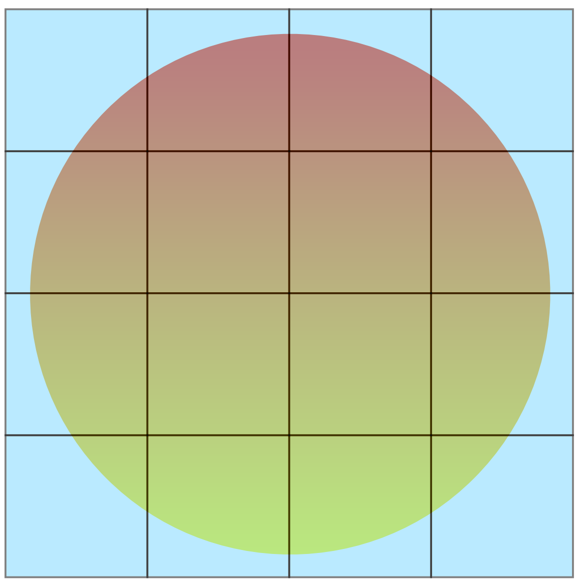


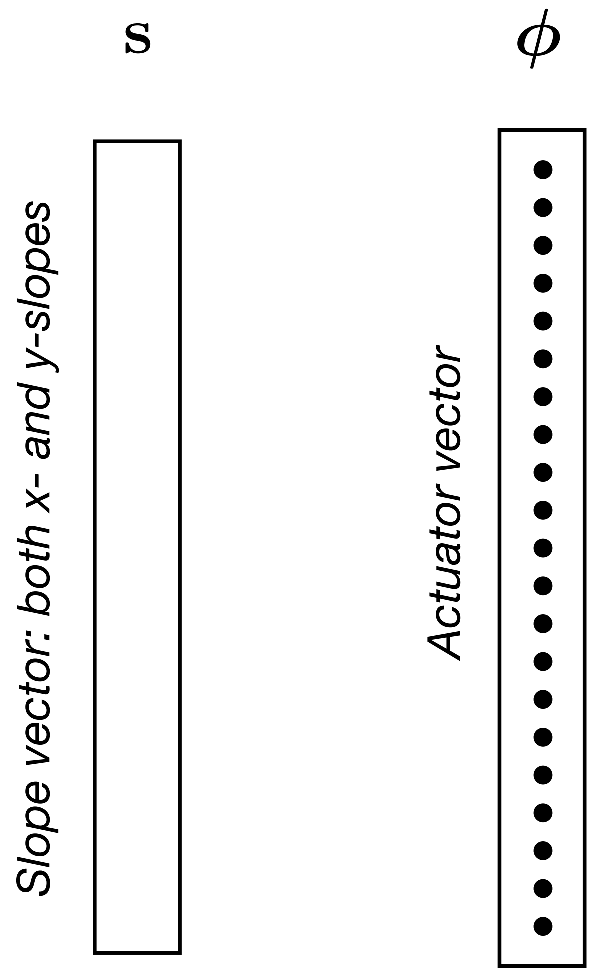
Phase in pupil



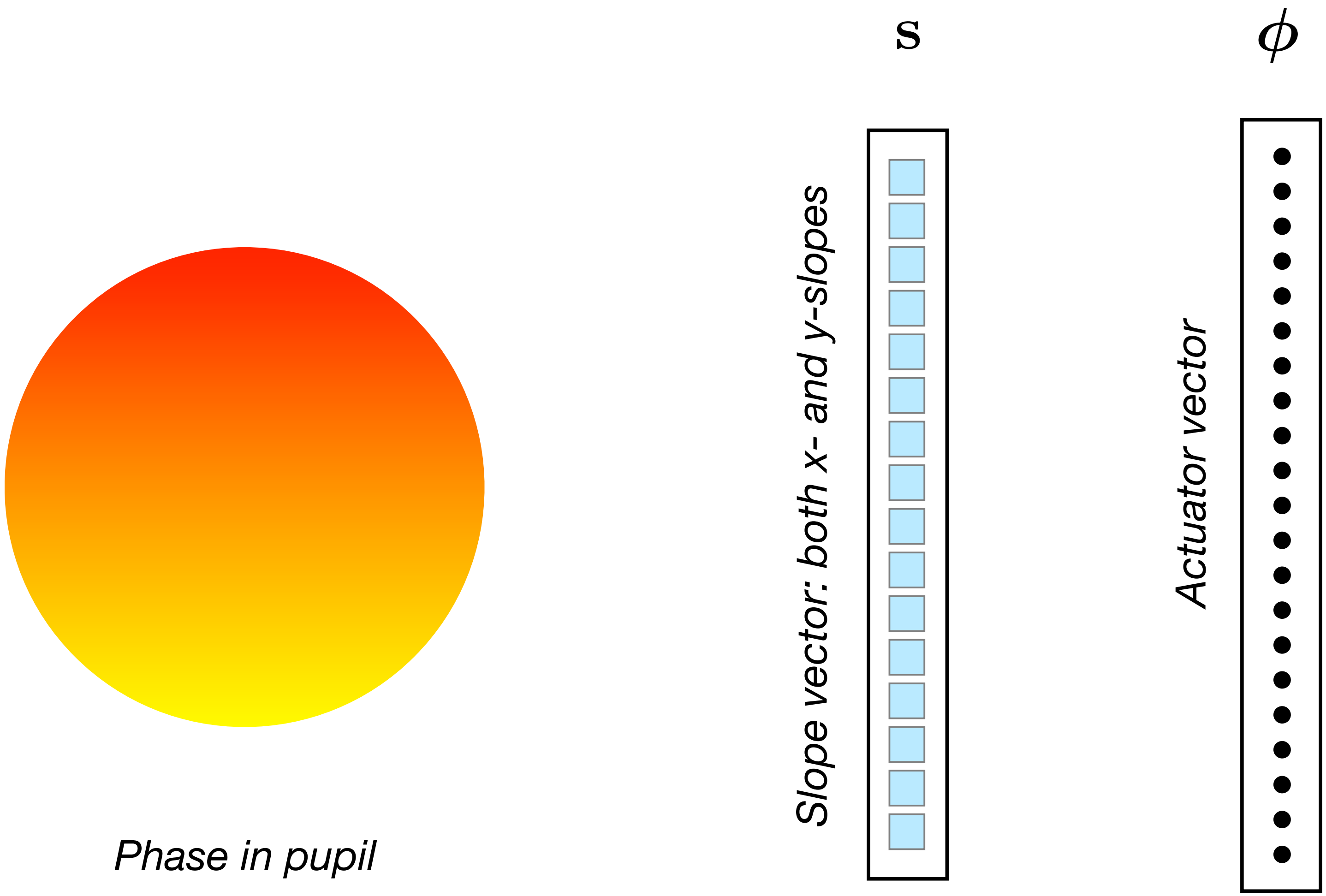
Mapping subapertures and actuators



Phase in pupil

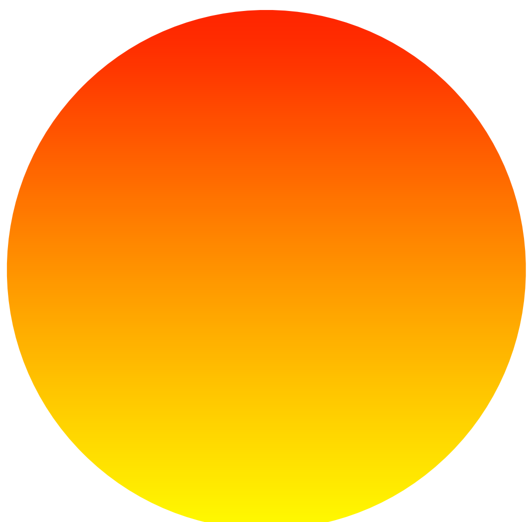


Mapping subapertures and actuators

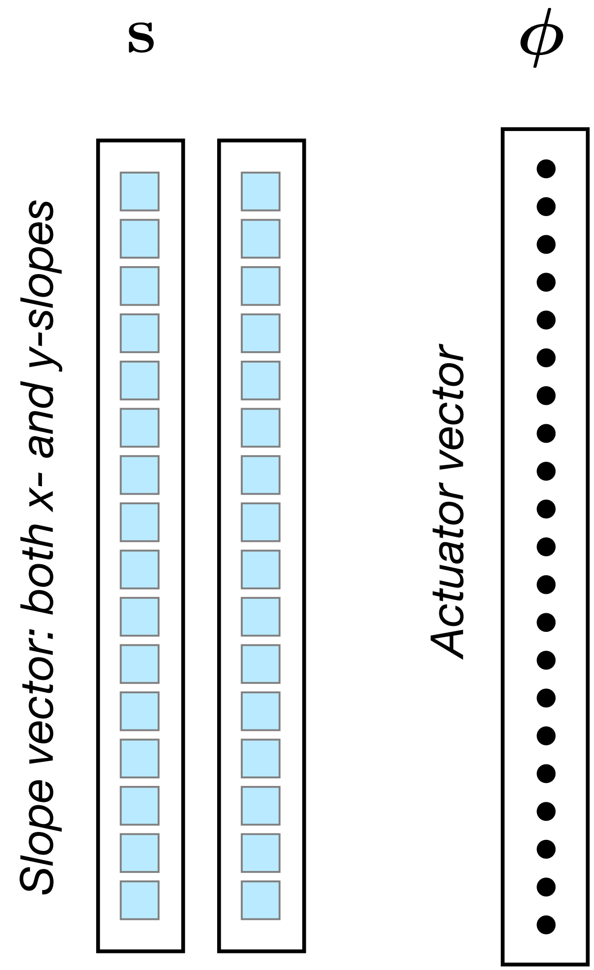


Phase in pupil

Mapping subapertures and actuators



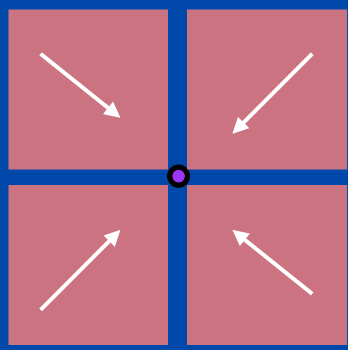
Phase in pupil



System matrix generation

- System matrix describes how a signal applied to the actuators, *a*, affects the WFS centroids, *s*.

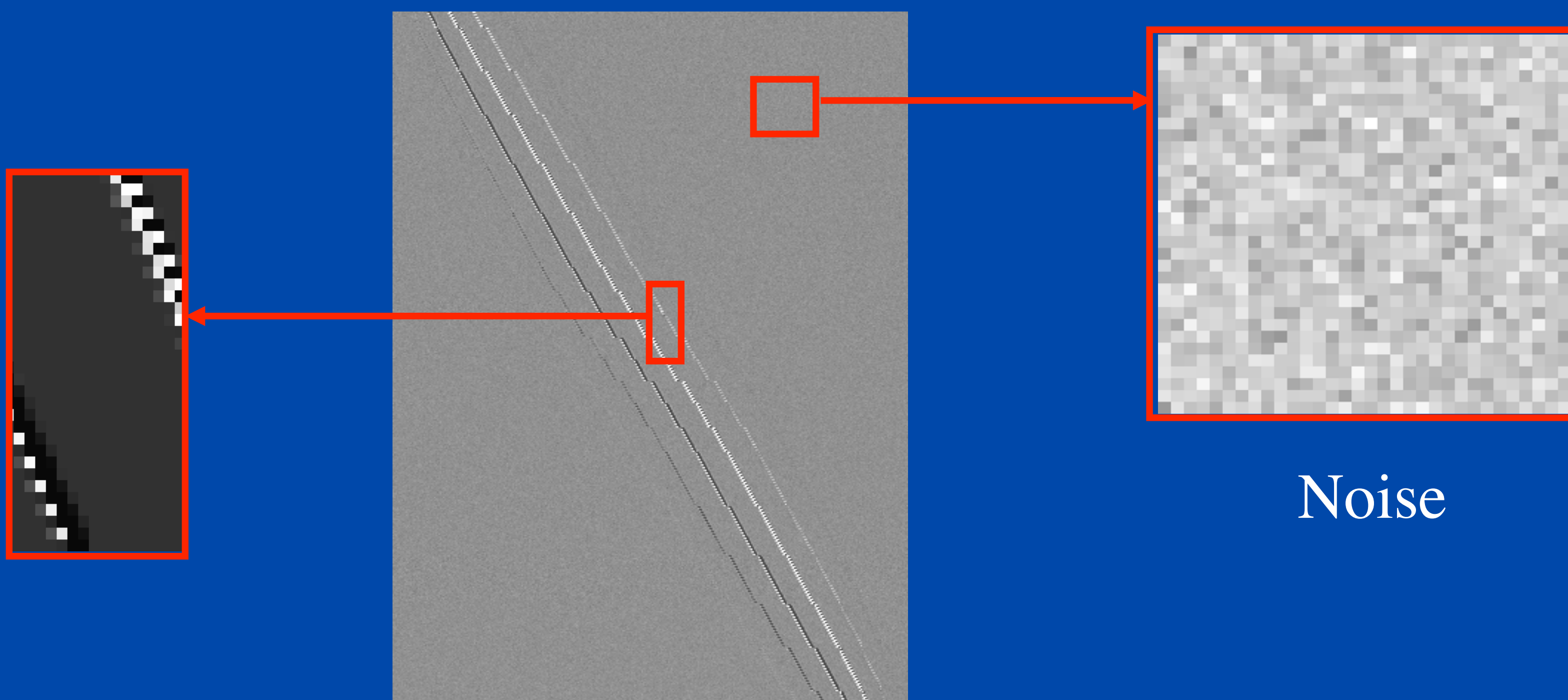
$$s = Ha$$



- Can be calculated theoretically or, preferably, measured experimentally

Experimental system matrix

- Poke one actuator at a time in the positive and negative directions and record the WFS centroids
- Set WFS centroid values from subapertures far away from the actuators to 0



Noise

System matrix

Inverting the system matrix

- We have the system matrix $s = Ha$
- We need a reconstructor matrix to convert from centroids to actuator voltages $a = Rs$

$$Ha = s$$

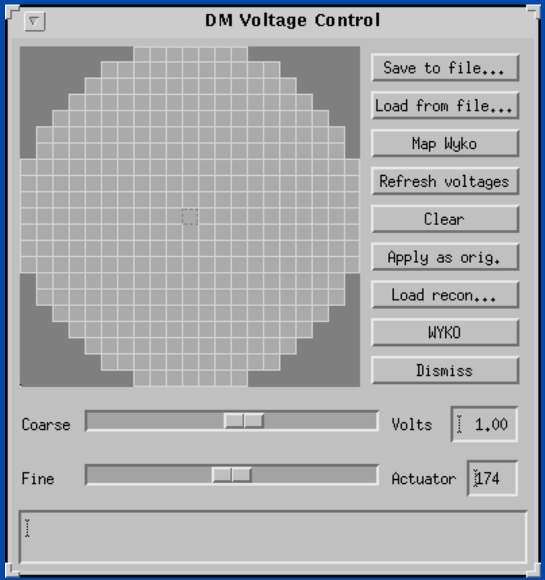
$$H^T Ha = H^T s$$

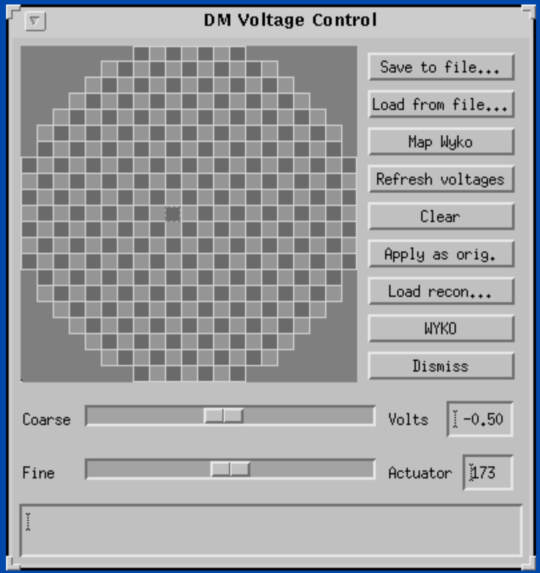
$$a = \underbrace{(H^T H)^{-1} H^T}_{R} s$$

Least-squares reconstructor

Least-squares reconstructor

- Least squares reconstructor is $(H^T H)^{-1} H^T$
- Minimizes $(s - Ha)^2$
- But $H^T H$ is not invertible because some modes are invisible!
- Two invisible modes are piston and waffle





Singular value decomposition (SVD)

- The SVD reconstructor is found by rejecting small singular values of ***H***.

- Write $H = U \Lambda V^T$

$$\Lambda = \begin{bmatrix} \lambda_1 & & & \\ & \lambda_2 & & \\ & & \lambda_3 & \\ & & & \ddots \end{bmatrix}$$

λ_i are the eigenvalues of $H^T H$

- The pseudo inverse is $H^+ = V \Lambda^{-1} U^T$

Singular value decomposition

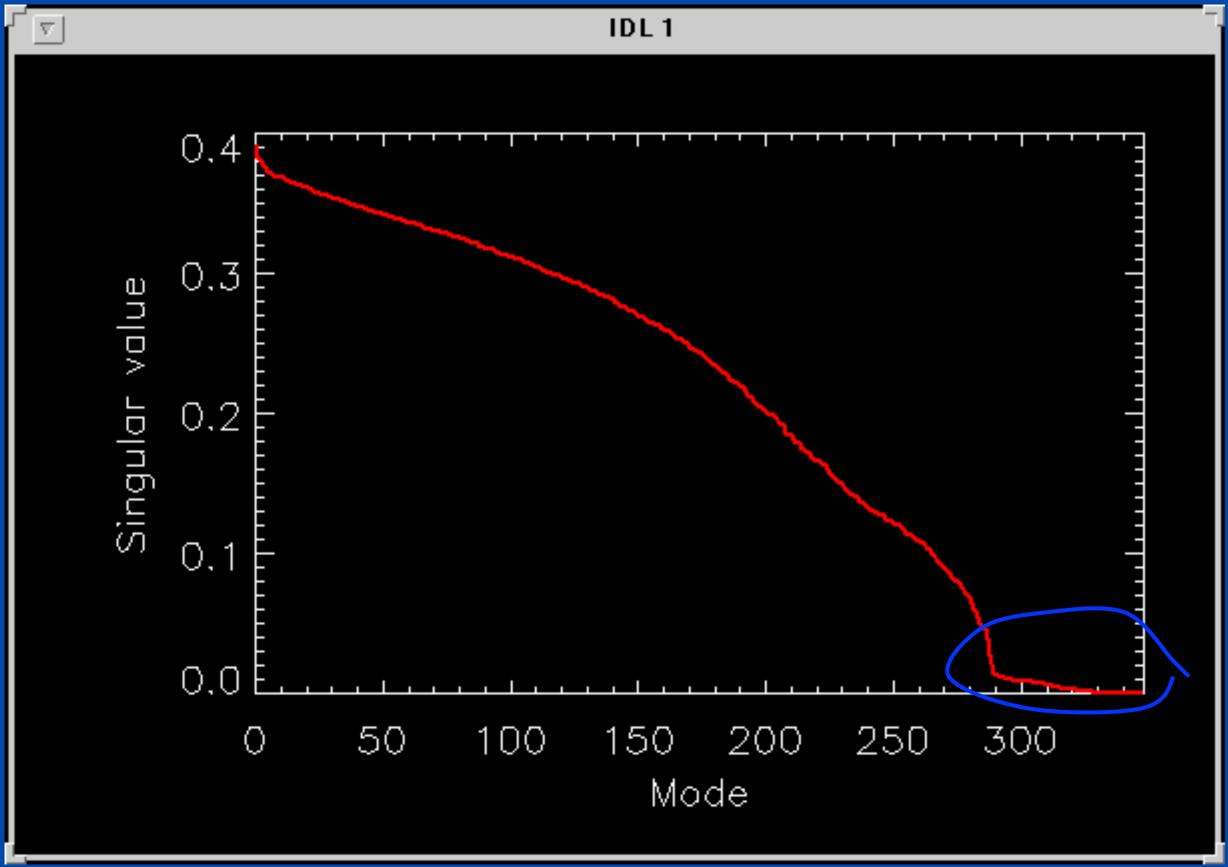
- The pseudo inverse is $H^{+} = V \Lambda^{-1} U^{T}$

$$\Lambda^{-1} = \begin{bmatrix} \lambda_1^{-1} & & & \\ & \lambda_2^{-1} & & \\ & & \lambda_3^{-1} & \\ & & & \ddots \end{bmatrix}$$

- Replace all the λ_i^{-1} with 0 for small values of λ_i

Singular value decomposition

- Example: Keck Observatory



← Set to zero

Noise propagation

- Suppose we only have centroid noise in the system with variance σ^2
- Variance of actuator commands is:

$$
\begin{aligned}
\mathrm{Var}(a) &= \mathrm{Var}(Rs) \\
&= \mathrm{E}[(Rs)^2] - \underbrace{(\mathrm{E}[(Rs)])^2}_{0} \\
&= \mathrm{E}[(Rs)^2] \\
&= |R|^2 \, \mathrm{E}[s^2] \\
&= |R|^2 \sigma^2
\end{aligned}
$$

Least-squares reconstructor

- For well-conditioned *H* matrices, we can penalize piston, *p*, and waffle, *w*:

$$p = [1,1,1,1,1,1,\ldots]^T$$

Invertible

$$w = [1,-1,1,-1,1,\ldots]^T$$

$$R = (H^T H + pp^T + ww^T)^{-1} H^T$$

- Minimizes $\underbrace{(s - Ha)^2}_{\text{Choose the actuator voltages that best cancel the measured centroids}} + (p^T a)^2 + (w^T a)^2$

Least-squares reconstructor

- **For well-conditioned *H* matrices, we can penalize piston, *p*, and waffle, *w*:**

$$p = [1,1,1,1,1,1,\ldots]^T$$

$$w = [1,-1,1,-1,1,\ldots]^T$$

$$R = (H^T H + pp^T + ww^T)^{-1} H^T$$

- **Minimizes** $(s - Ha)^2 + \underbrace{(p^T a)^2}_{} + (w^T a)^2$

Choose the actuator voltages such that there is no piston

Least-squares reconstructor

- For well-conditioned *H* matrices, we can penalize piston, *p*, and waffle, *w*:

$$p = [1,1,1,1,1,1,\ldots]^T$$

$$w = [1,-1,1,-1,1,\ldots]^T$$

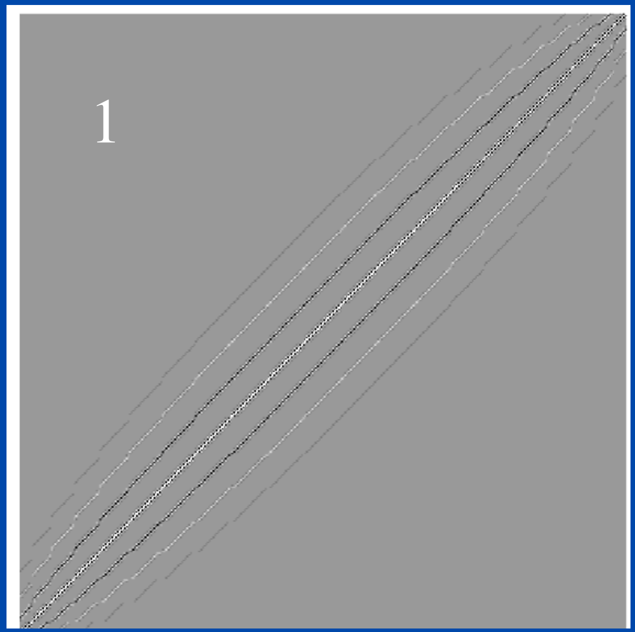
$$R = (H^T H + pp^T + ww^T)^{-1} H^T$$

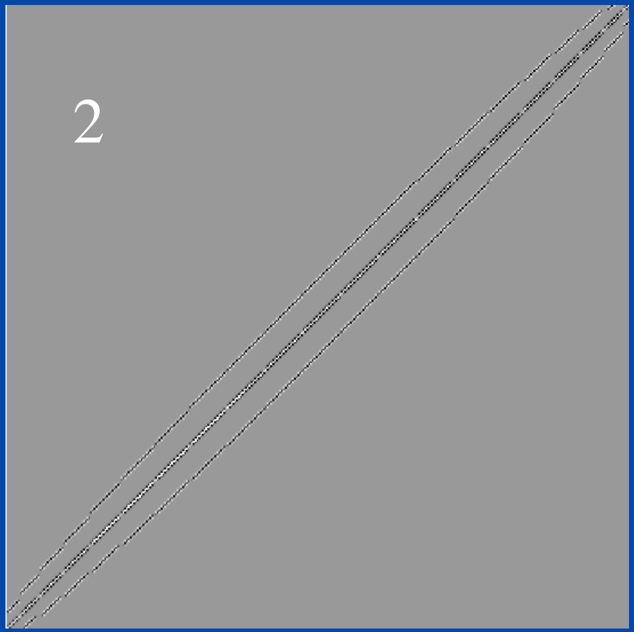
- Minimizes $(s - Ha)^2 + (p^T a)^2 + \underbrace{(w^T a)^2}_{}$

Choose the actuator voltages such that there is no waffle

Least-squares reconstructor

- Penalize waffle in the inversion:
  1. Inverse covariance matrix of Kolmogorov turbulence or
  2. Waffle penalization matrix

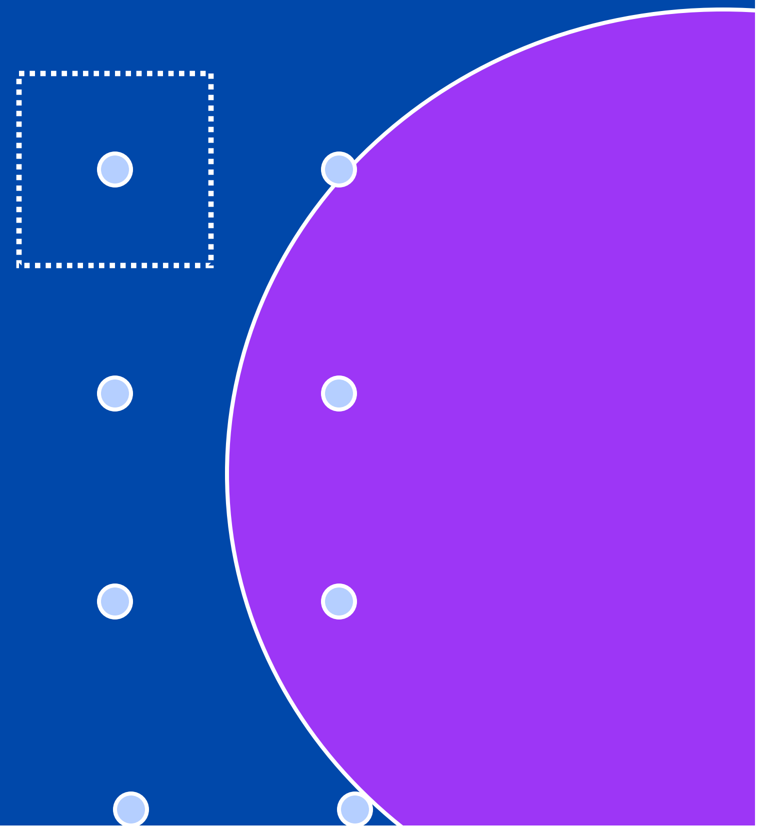




Slaved actuators

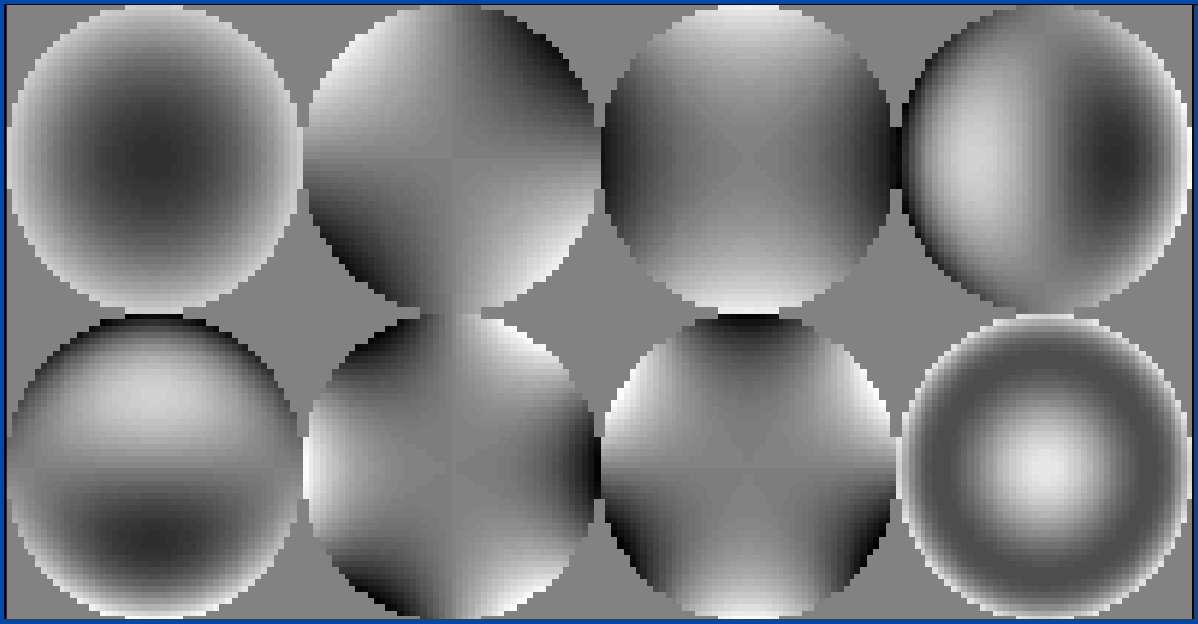
- Some actuators are located outside the pupil and do not directly affect the wavefront
- They are often "slaved" to the average value of its neighbors

Slaved to average value of its neighbors



Modal reconstructors

- Can choose to only reconstruct certain modes
- Avoids reconstructing unwanted modes (e.g., waffle)



Zernike modes

Modal reconstructors

$Z = [z_1, z_2, z_{3,} \ldots]$ Zernike modes

HZ Centroids measured by applying Zernike modes to the DM

$R = Z[(HZ)^T (HZ)]^{-1} (HZ)^T$ Zernike reconstructor

Hardware approaches

- Systems until recently could use fast CPUs
- For advanced AO systems, need to use more processors and be able to split the problem into parallel blocks
- GPU - Graphics Processing Unit
- DSP - Digital Signal Processor
- FPGA - Field Programmable Gate Array (lots and lots of logic gates)